Course syllabus
"Fundamental problems of quantum mechanics"

Lecturer: Prof. Volodymyr Tkachuk [5 ECTS credits]

I. Mathematical foundations of quantum mechanics

1. State space.
2. Operators.
3. Representation of the state vectors and operators.
4. Properties of eigenvalues and eigenvectors of Hermitian operators.
5. Unitary transformations.
6. Postulate of measurement in quantum mechanics.
7. Mean value of physical quantities. Pure-state ensemble.
8. Mixed-state ensemble. Density matrix.
9. Quantum Poisson bracket.
10. Dynamics of quantum system.
11. Dynamics of quantum system. Mixed-state ensemble.
12. Heisenberg uncertainty relation.
13. Measurement in pure-state and mixed-state ensembles.

II. Two state quantum systems

1. Quantum states of spin-1/2 particle.
2. Spin-density matrix.
3. Quantum states of N spins.
4. Singlet state. EPR paradox.
5. Bell's inequalities.
6. Entangled states. Measure of entanglement.
7. GHZ equality.

III. Quantum communications

1. No-cloning theorem.
2. Quantum teleportation.

3. Quantum cryptography.

IV. Quantum computing and quantum computers

1. From classical bit to quantum bit.
2. Quantum processor. Quantum gates.
3. Quantum algorithms. The Deutsch–Jozsa problem.
4. Physical implementation of single-qubit gates
5. Single-qubit quantum computer.

V. Measurement in quantum mechanics

1. Measurement of the projection of spin on a given direction.
2. Stern–Gerlach experiment.
3. Is it possible to measure the quantum state?
4. Measurement without interaction.

VI. Geometry of quantum state space

1. Distance between quantum states.
2. Metric of quantum state space.
3. Metric of two-dimensional quantum space.

VII. Evolution of a quantum system

1. Speed of quantum evolution.
2. Quantum brachistochrone problem.
3. Zeno effect.
4. Adiabatic theorem.
5. Berry phase.

VIII. Decoherence

1. Spin in a fluctuating magnetic field.
2. Exact model of decoherence.
3. Decoherence of Schrödinger's cat.

IX. Operator identity and mean value of functions of bosonic operators

1. Identities for functions of bosonic operators.
2. Parameter derivative.
3. Weyl's identity.
4. Mean values of functions of bosonic operators.