IVAN FRANKO NATIONAL UNIVERSITY OF LVIV

WORKSHOP ON CURRENT PROBLEMS IN PHYSICS

PROGRAM AND ABSTRACTS

Lviv, 05–07 July 2016

05 July 2016

09:00–09:30 Registration and opening
(8, Kyryla i Mefodia St., "Velyka Fizychna" Lecture Hall)

Chairman: **V. Tkachuk**

09:30–10:00 *A. Drzewiński, D. Woźniak*, Dynamics of the spin-1 Heisenberg antiferromagnetic chain after a quantum quench

10:00–10:30 *Ch. von Ferber, Yu. Holovatch*, Variety of scaling laws for DNA denaturation

10:30–11:00 *Yu. Sitenko*, Chiral effects in a slab of hot dense magnetized spinor matter

11:00–11:30 Coffee break

Chairman: **A. Gavrilik**

11:30–12:00 *A. Rovenchak*, Physics at the University of Lviv: some interesting facts

12:00–12:15 *A. Barasiński*, Tripartite entanglement versus tripartite nonlocality

12:15–12:30 *P. Sarkanych, Yu. Holovatch, R. Kenna*, 1D Potts model with invisible states

12:30–12:45 *U. Liudkevych, I. Shtablavyi, S. Mudry*, Thermal expansion of liquid metals

Lunch

Chairman: **Yu. Sitenko**

14:30–15:00 *S. Kondej*, Geometry of quantum system and its spectral properties

15:00–15:30 *B. Brzostowski, D. Sztolberg, P. Dereń, B. Bondzior, W. Walerczyk, G. Banach*, Impact of point defects on electronic and spectroscopic properties of phosphors

15:30–15:45 *R. Neomenko, B. Novosyadlyj*, The impact of dark energy-dark matter interaction on the expansion dynamics of Universe

15:45–16:00 *K. Rożko, J. Kijak, W. Lewandowski, R. Basu, G. Melikidze*, The study of two special pulsar's spectra: J1740+1000 and B1800-21

16:00–16:15 *O. Kalyuzhnyi, J. Ilnytskyi, C. von Ferber, Yu. Holovatch*, Universal shape properties of mesoscopic polymer chains and polymer stars

16:30 Concert

06 July 2016

Chairman: **A. Drzewiński**

09:30–10:00 *A. Gavrilik*, Higher order distributions and correlations in the two-parameter deformed Bose gas models

10:00–10:30 *V. Tkachuk*, Soccer-ball problem in the space with minimal length

10:30–11:00 *B. Padlyak*, Potential applications of new functional materials based on the borate compounds

11:00–11:30 Coffee break

Chairman: **Yu. Holovatch**

11:30–12:00 *J. Kalaga, W. Leoński*, Quantum steering generation in a three-qubit systems

12:00–12:15 *Yu. Mishchenko, A. Gavrilik*, Mapping of composite fermi-particles into fermions in many-body systems, and its applications

12:15–12:30 *Kh. Gnatenko*, System of two particles in rotationally invariant noncommutative space

12:30–12:45 *M. Marć, M. R. Dudek*, Magnetically modified titanate nanotubes for water purification

Lunch

Chairman: **B. Padlyak**

14:30–15:00 *B. Novosyadlyj, M. Tsizh*, Voids as a test for dark energy models

15:00–15:30 *Yu. Yaremko, M. Przybylska, A. J. Maciejewski*, Penning trap with an inclined magnetic field

15:30–15:45 *A. Kuzmak*, Entanglement of spin chain with long-range interaction

15:45–16:00 *A. Karczewska, P. Rozmej, Ł. Rutkowski*, Nonlinear wave equation for shallow water problem with nonflat bottom of the container

16:00–16:30 Coffee break

Chairman: **A. Rovenchak**

16:30–16:45 *M. Tsizh, B. Novosyadlyj, Yu. Kulinich*, Dependence of the evolution of 3-component superclaster size Halo on dark energy parameters

16:45–17:00 *M. Samar*, Singular potentials in general case of deformed space with minimal length

17:00–17:15 *W. Szumiński*, Two simple pendula interacting harmonically

17:15–17:30 *D. Sztolberg, B. Brzostowski, P. Dereń*, $LaAlO_3$:Tm^{3+} spectroscopic properties dependency of crystal internal structure

07 July 2016

Chairman: **B. Novosyadlyj**

09:30–10:00 *W. Lewandowski*, Probing the ionized interstellar medium using radio pulsar signals

10:00–10:30 *M. Stodilka, A. Sukhorukov*, Jets in the quiet solar photosphere

10:30–11:00 *V. Myhal, O. Derzhko*, Wetting in presence of electric field: Classical-density-functional-theory study for model system

11:00–11:30 Coffee break

Chairman: **Yu. Yaremko**

11:30–11:45 *M. Stetsko*, Squashed topological Kaluza-Klein black hole solution

11:45–12:00 *Y. Humenyuk*, Thermodynamic quantities of the low-density gas in the weakly nonequilibrium heat-conduction steady state in the linear temperature profile approximation

12:00–12:15 *V. Vasyuta*, Field equations in a space with spin noncommutativity of coordinates

12:15–12:30 *M. Szczeciński*, On stochastic second order Korteweg - de Vries type equations

12:30–12:45 *O. Sergijenko, B. Novosyadlyj*, B-mode polarization from the primordial gravitational waves: theory and observations

12:45 Closing

14:00 Informal meeting

DYNAMICS OF THE SPIN-1 HEISENBERG ANTIFERROMAGNETIC CHAIN AFTER A QUANTUM QUENCH

A. Drzewiński, D. Woźniak
Institute of Physics, University of Zielona Góra, Poland

In the framework of the Matrix Product State representation the effect of a sudden turning on of the uniaxial anisotropy on the time evolution of the Haldane state has been investigated. Depending on the value of the uniaxial anisotropy parameter, the calculations were derived within (or outside) the region where the Haldane gap survives. An exact expression for the time evolution of the Loschmidt echo has been derived and, moreover, its collapse and revival behaviour was captured. In addition, the non-local order parameter of a time evolving state was tracked, revealing two types of relaxations.

VARIETY OF SCALING LAWS FOR DNA DENATURATION

Ch. von Ferber[1,2,4]*, Yu. Holovatch*[3,4]
[1]Applied Mathematics Research Centre, Coventry University, United Kingdom,
[2]Heinrich-Heine Universität Düsseldorf, Germany,
[3]Institute for Condensed Matter Physics, NAS of Ukraine, Lviv
[4]Doctoral College for the Statistical Physics of Complex Systems,
Leipzig-Lorraine-Lviv-Coventry ($\mathbb{L}^4$), Leipzig, Germany

Here, we rediscuss the thermal denaturation transition of DNA in a solvent: as the temperature is increased the DNA double strand separates into two individual single strands in an unbinding transition (also called denaturation transition). The discussion of the order of this transition has a long history ever since a minimal model was suggested by Poland and Scheraga. There are numerous studies devoted to determining the order of the transition between denaturated and bound DNA states. We briefly review some of the approaches and further discuss in detail the effects of the environment (quality of the solution, disorder) that may impact on the order of the transition. These show up in changes of the scaling exponents governing the conformational properties of DNA strands.

To this end, we re-consider the Poland-Scheraga model and apply a polymer field theory approach to calculate entropic exponents associated with the denaturated loop distribution. We discuss in particular variants of this transition that may occur due to the properties of the solution and that affect the self- and mutual interaction of both single and double strands.

We show that different environments may shift the transition towards or away from the first order regime. We find that the effects studied significantly influence the strength of the first order transition. This becomes manifest in the changes shown by the scaling laws that govern the DNA loop and strand distribution. As a quantitative measure of these changes we present the values of the corresponding scaling exponents. In the 2D case exact exponents may be deduced, mapping the polymer model onto a two-dimensional random lattice modelling quantum gravity. For the 3D case we derive an expansion in $4 - \varepsilon$ dimensions and evaluate the perturbation series to ε^4 by means of resummation techniques.

This work was supported by the Seventh Framework Program of the European Union within the International Research Staff Exchange Scheme Grants Nos 612669 *Structure and Evolution of Complex Systems with Applications in Physics and Life Sciences* and 612707 *Dynamics of and in Complex Systems.*

CHIRAL EFFECTS IN A SLAB OF HOT DENSE MAGNETIZED SPINOR MATTER

Yurii Sitenko
Bogolyubov Institute for Theoretical Physics, Kyiv

We study an influence of boundaries on chiral effects in hot dense relativistic spinor matter in a strong magnetic field which is transverse to bounding planes. The most general set of boundary conditions ensuring the confinement of matter within the bounding planes is considered. We find that, in thermal equilibrium, the nondissipative axial current along the magnetic field is induced, depending on chemical potential and temperature, as well as on a choice of the boundary condition. As temperature increases from zero to large values, a stepped-shape behaviour of the axial current density as a function of chemical potential is changed to a smooth one; the choice of the boundary condition can facilitate either amplification or diminution of the chiral separation effect. This points at a significant role of boundaries for physical systems with hot dense magnetized relativistic spinor matter, i.e. compact stars, heavy-ion collisions, novel materials known as the Dirac and Weyl semimetals.

[1] Yu.A. Sitenko, J. Phys.: Conf. Series. **670**, 012048 (2016).
[2] Yu.A. Sitenko, E-print arXiv:1603.09268, 8 March (2016).

PHYSICS AT THE UNIVERSITY OF LVIV: SOME INTERESTING FACTS

A. Rovenchak
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

First accounts related to physics at the University of Lviv refer to the second half of the 17th century. The manuscript by Mikołaj Rossnowski entitled *Physica, siue scientia naturalis...* appeared in 1668, just seven years after the foundation of the University in 1661. Some interesting facts in the history of this discipline at the University of Lviv will be discussed. In particular, we will mention activities of Ignác Martinovics, the first Professor of Physics at this institution (1784–91). Among others, the works by Wojciech Urbański will be analyzed as he is likely the first person habilitated in the field of mathematical physics (1850) in the Habsburg Empire. In 1872–73, the Chair of Physics was split into two, Experimental Physics and Mathematical (Theoretical) Physics. The famous representative of the latter is Marian Smoluchowski who spent in Lviv his most productive years (1899–1913). We will finish the presentation by the Interbellum period, in particular referring to Lepold Infeld, leaving subsequent developments in the last seven decades for a separate consideration.

TRIPARTITE ENTANGLEMENT VERSUS TRIPARTITE NONLOCALITY

A. Barasiński
Quantum Optics and Engineering Division, Institute of Physics, University of Zielona Góra, Poland

Quantum entanglement and nonlocality are two special aspects of quantum correlations. The relationship between multipartite entanglement and nonlocality is at the root of the foundations of quantum mechanics for which there is still no general quantitative theory. In order to answer this question we analyze the relationship between two kinds of genuine tripartite entanglement measures that are considered as the three-qubit analogs of the concurrence and negativity. We describe the states which are extremal in one measure for a given value of the second measure. Those states can serve as "waymarks" for three-qubit entangled states providing means for their systematic classification. Moreover, we show the close relation between those extremal states and the maximal possible violation of the Bell-type inequalities for general three-qubit pure states. Finally, we derive an analytical expression relating tripartite entanglement to the maximal violations of the Bell-type inequalities.

1D POTTS MODEL WITH INVISIBLE STATES

P. Sarkanych[1,2], *Yu. Holovatch*[1,3], *R. Kenna*[2,3]
[1]Institute for Condensed Matter Physics, NAS of Ukraine, Lviv
[2]Applied Mathematics Research Centre, Coventry University, United Kingdom
[3]Doctoral College for the Statistical Physics of Complex Systems,
Leipzig-Lorraine-Lviv-Coventry ($\mathbb{L}^4$), Leipzig, Germany

We present an exact solution of the 1D Potts model with invisible states. The model was introduced a few years ago to explain some untypical phase transitions with spontaneous symmetry breaking. In addition to q ordinary Potts states this model possesses r states which do not interact, and thus contribute to the entropy, but not to the internal energy. The number of invisible states plays a role of a parameter, which changes the order of a phase transition. We consider the case of two ordering fields: h_1 acting on the first visible state and h_2 on the first invisible state. Using transfer matrix method we obtain the partition function of the model and further analyse partition function zeros in the complex T and h planes (Fisher and Lee–Yang zeros). We find the locus of Yang-Lee edge and generalize the duality transformation which maps Lee-Yang zeros to Fisher zeros for $r > 0$. At $h_1 = 0$ and $\mathrm{Im}\, h_2 = 0$ Fisher zeros accumulate along the line that intersects real T-axis at $T = 0$. This corresponds to the usual phase transition in a 1D system. However, for the $r < 0$ or $\mathrm{Im}\, h_2 \neq 0$ the line of zeros intersects the positive part of the real T-axis, which means an existence of a phase transition at non-zero temperature.

THERMAL EXPANSION OF LIQUID METALS

U. Liudkevych, I. Shtablavyi, S. Mudry
Department for Metal Physics, Ivan Franko National University of Lviv, Ukraine

Experimental measurements of thermal expansion coefficient for liquid metals are related with more problems in comparing with crystalline ones. Available results on density for liquid metals allowed establishing principal features of thermal expansion in liquid metals over wide temperature range. Unfortunately there are only few papers, where the relation between short range order structure and thermal expansion mechanism is analyzed. It is also still unclear how the cluster structure affects the mechanism of thermal expansion as well as free volume.

On that reason the aim of this work was to study the temperature dependences of main structure parameters, obtained from XRD- measurements.

These parameters, first of all such as mean interatomic distance, most probable interatomic distance, number of neighbors and size of structural units have been analyzed and used for calculation of thermal expansion coefficient.

Liquid In and Bi have been studied by means of X-ray diffraction method over wide temperature range. Some of temperature dependences of structure parameters, obtained from diffraction data reveal the deviation from linear function supposing the existence of significant changes of short range order structure at any definite temperatures. More sensitive to temperature is the mean intewratomic distance in comparasion to most probable one, which almost linearly decreases with temperature. The size of first coordination sphere determined as first sharp minimum after principal peak in pair correlation function increases in fact linearly for liquid In and shows the maximum point at about 1000 K for molten Bi. At the same time the number of nearest neighbors for liquid In decreases linearly, whereas the similar dependence for liquid Bi has a maximum at about 900 K. Such behavior is in accordance with structure features of both liquid metals. Indium in liquid state has a less inhomogeneous structure in liquid state comparing to liquid Bi and on that reason the structure changes in last of them is more pronounced.

GEOMETRY OF QUANTUM SYSTEM AND ITS SPECTRAL PROPERTIES

S. Kondej
Institute of Physics, Faculty of Physics and Astronomy, University of Zielona Góra, Polska

We discuss a relation between geometry of certain class of quantum systems and spectral properties of particles living in these systems. In particular, we are interested in quantum wires and waveguides. Our aim is to give a mathematical description of such structures and analyze discrete spectrum induced by geometry as well as existence and properties of resonances.

IMPACT OF POINT DEFECTS ON ELECTRONIC AND SPECTROSCOPIC PROPERTIES OF PHOSPHORS

B. Brzostowski[1], *D. Sztolberg*[1], *P. Deren*[2], *B. Bondzior*[2], *W. Walerczyk*[2], *G. Banach*[3]
[1]Institute of Physics, University of Zielona Góra, Poland
[2]Institute of Low Temperatures and Structure Researches, PAS, Wrocław, Poland
[3]DATAX Sp. z o.o., Wrocław, Poland

The role of vacancies in the determination of the levels of impurities in the band gap is examined within the framework of density functional formalism (DFT). The presence of vacancies influences the density of states in a very strong manner. Theoretical results for different materials (perovskites, aluminum silicates and phosphates) doped with the transition metal or rare earth ions are presented and compared with the results of the experiment.

The results of the calculations allow for correct interpretation and full understanding of the results of experiments. Cation or oxygen vacancy in a host material lattice is discussed.

THE IMPACT OF DARK ENERGY-DARK MATTER INTERACTION ON THE EXPANSION DYNAMICS OF UNIVERSE

R. Neomenko, B. Novosyadlyj
Astronomical observatory, Ivan Franko National University of Lviv, Ukraine

We analyze the dynamics of expansion of homogeneous and isotropic Universe in the model of dynamical dark energy which interacts with dark matter. The interactions with baryon and other observable kinds of matter we consider to be very small or to be absent. All components of Universe are described by approximation of ideal fluid. As the base model of dark energy we took the model in which parameter of equation of state of dark energy is dependent on time and is parameterized by adiabatic sound speed. Using expressions for energy densities for dark components obtained from general covariant conservation equations was shown that exists the regions in values of interaction parameter which characterize the strength of interaction and parameters of dark components for which energy densities of dark energy and dark matter can take the negative value. From conditions of positivity of energy densities of dark components the constraints on interaction parameter were obtained.

THE STUDY OF TWO SPECIAL PULSAR'S SPECTRA: J1740+1000 AND B1800-21

K. Rożko, J. Kijak, W. Lewandowski, R. Basu, G. Melikidze
Janusz Gil Institute of Astronomy, University of Zielona Góra, Polska

In my talk I would like to discuss two cases of unusual spectra of pulsars: J1740+1000 and B1800-21. In both cases the recent measurements have brought results that do not allow for unambiguous interpretation of the shape of their spectra. To solve this problem I have developed an observational project that will be conducted using two of the biggest radio telescopes in the world: the Giant Metrewave Radio Telescope (located in India) and The Green Bank Telescope (in USA). That project is aimed at the determination of the actual shape of both pulsar's spectra.

UNIVERSAL SHAPE PROPERTIES OF MESOSCOPIC POLYMER CHAINS AND POLYMER STARS

O. Kalyuzhnyi[1,2], *J. Ilnytskyi*[1,2], *C. von Ferber*[3,2], *Yu. Holovatch*[1,2]
[1] *Institute for Condensed Matter Physics, NAS of Ukraine, Lviv* [2] *Doctoral College for the Statistical Physics of Complex Systems, Leipzig-Lorraine-Lviv-Coventry (L^4)* [3] *Applied Mathematics Research Centre, Coventry University, United Kingdom*

We study shape characteristics, such as mean prolateness, asphericity and size ratio, of long flexible polymer chains and star-shape polymers in solvents of varying quality. These properties are universal and shared by polymers of different chemical content depending on the space dimension d only. So far, the shape characteristics have been analyzed using mainly the self avoiding walk representation of polymer chains on a lattice, whereas the off-lattice simulations are more scarce. In this study we employ the dissipative particle dynamics simulations in $3D$.

For the case of linear chains in a good solvent, we see that the distributions for all shape characteristics to be rather broad [1], but their average values to agree well with the renormalization group results [2,3,4] and Monte Carlo simulations of lattice models [5,6].

The homo-star polymers are considered in a solvent of variable quality. The changes undergone by the shape characteristics with varying solvent quality are in a good agreement with available data of previous studies [7]. In addition, we discuss a broader range of such characteristics and comment on their maxima observed near the θ-point of a solvent [8].

The hetero-stars, characterised by variable solubility for individual arms, are also studied [8]. Various combinations of arms solubility are considered leading to specific conformations and affecting the shape characteristics of these molecules. We see such systems as the micellogenes with tunable aggregation abilities.

[1] O. Kalyuzhnyi, Ja. Ilnytskyi, Yu. Holovatch. J. Phys. Stud.**18** (2014) 4602.
[2] O. Jagodzinski, E. Eisenriegler, K. Kremer, J. Phys.I(France) **2** (1992) 2243.
[3] J.A. Aronovitz, D.R. Nelson, J. Physique **47** (1986) 1445.
[4] M. Behamou, G. Mahoux, J. Physique Lett. **46** (1985) L689.
[5] O. Jagodzinski, E. Eisenriegler, K. Kremer, J. Phys. I (France) **2** (1992) 2243.
[6] M. Bishop, C.J. Saltiel, J. Chem. Phys. **88** (1988) 6594.
[7] M. M. Nardai, G. Zifferer, J. Chem. Phys. **131** (2009) 124903.
[8] O. Kalyuzhnyi, Ja. Ilnytskyi, C. von Ferber, Yu. Holovatch. *in preparation.*

HIGHER ORDER DISTRIBUTIONS AND CORRELATIONS IN THE TWO-PARAMETER DEFORMED BOSE GAS MODELS

A. M. Gavrilik
Bogolyubov Institute for Theoretical Physics of the NAS of Ukraine, Kyiv

In this talk I give a concise review of the explicit results on two-, three-particle and higher order (r-particle) momentum distributions and correlation function intercepts in some 2-parameter deformed extensions of the Bose gas model. In all the explored deformed Bose gas models, the obtained explicit formulas are exact, being valid not only for small extent of deformation (as measured by the deviation $|1-q|$ in some one-parameter deformed models). Due to that, qualitatively new effects may appear when large deviations from the non-deformed case (large deformations) are explored. Besides the popular p,q-deformed model (recall that p,q-deformed oscillator or p,q-bosons have been introduced [1] yet in 1991, and general relevant explicit formulas for p,q-Bose gas model derived in [2]), quite recently yet another two-parameter $(\tilde{\mu},q)$-deformed Bose gas model has been developed [3] with the aim of taking effectively into account compositeness of particles jointly with their interactions. Earlier, a realization of the operator algebra of composite bosons in terms of the algebra of deformed oscillator

(DO) quadratically nonlinear in the particle number operator N was proven, with the deformation (or nonlinearity) parameter $\tilde{\mu}$. Then, close connection has been found [4] of the inter-component bipartite entanglement (notion important in quantum information theory) of quasi-bosons on one hand, and the deformation parameter $\tilde{\mu}$ of the DO on the other. Energy dependence of the entanglement entropy in different states of quasi-bosons was also studied.

What concerns thermodynamics of that model, the $(\tilde{\mu}, q)$-deformed virial expansion of the equation of state (EOS) was obtained [5] along with the first five virial coefficient. More detailed analysis of the second virial coefficient of $(\tilde{\mu}, q)$-Bose gas model has shown direct relation of q and $\tilde{\mu}$ with the parameters responsible resp. for interaction and compositeness.

Main part of the talk is devoted to the applied aspects of $(\tilde{\mu}, q)$-Bose gas model – the (deformed) distributions and correlation functions. We derive explicitly the one- and two-particle distribution functions, and the intercept of two-particle momentum correlation function [6]. The obtained results are (i) compared with the existing results in another deformed analogs of the Bose gas model, and (ii) applied to the system of pions generated and registered in the RHIC/STAR experiments on relativistic heavy-ion collisions. When our theoretical curves are compared with the experimental data on $\pi^{\pm}\pi^{\pm}$ two-pion correlation function intercepts, we find nice agreement [6]. From this successful use of $(\tilde{\mu}, q)$-Bose gas model to describe the properties of real pions in the RHIC/STAR experiments we conclude: both the compositeness of those particles and their interaction play essential role.

[1] R. Chakrabarti, R. Jagannathan, J. Phys. A **24** (1991), L711.
[2] L. V. Adamska, A. M. Gavrilik, J. Phys. A: Math. Gen. **37** (2004), 4787.
[3] A. M. Gavrilik, Yu. A. Mishchenko, Ukr. J. Phys. **58** (2013), 1171.
[4] A. M. Gavrilik, Yu. A. Mishchenko, Phys. Lett. A **376** (2012), 1596.
[5] A. M. Gavrilik, Yu. A. Mishchenko, Phys. Rev. E **90** (2014), 052147.
[6] A. M. Gavrilik, Yu. Mishchenko, Nucl. Phys. B **891** (2015), 466.

SOCCER-BALL PROBLEM IN THE SPACE WITH MINIMAL LENGTH

V. M. Tkachuk
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

Description of macroscopic bodies in a quantized space (space with minimal length) is called the Soccer-Ball problem. Macroscopic bodies consist of elementary particles and let us suppose we know the behavior these elementary particles in the quantized space. Then we have a question about the behavior of the center of mass of macroscopic bodies in the quantized space. With this problem are related such problems as equivalence principle, properties of kinetic energy, Galilean and Lorenz transformation, etc. in a space with minimal length. Just this question will be discussed in this talk.

POTENTIAL APPLICATIONS OF NEW FUNCTIONAL MATERIALS BASED ON THE BORATE COMPOUNDS

B. V. Padlyak[1,2]
[1]University of Zielona Gŕa, Institute of Physics, Zielona Gŕa, Poland [2]Vlokh Institute of Physical Optics, Lviv, Ukraine

Borate crystals and glasses represent suitable host materials for many useful practical applications. In particular, the borate compounds, un-doped and doped with rare-earth and transition elements, are very promising materials for nonlinear optics, quantum electronics including laser technique, scintillators and thermoluminescent dosimeters, detectors and transformers of the ionising radiation, ultraviolet–visible converters and other applications.

In this report are presented the review of publications, including our recent articles [1–5], devoted to study of the spectroscopic properties of a series of un-doped as well as transitional and rare-earth

elements doped borate glasses and crystals. Possibilities of applications of the borate crystalline and glassy compounds for harmonic transformers, active elements of the solid-state lasers, ultraviolet–visible convertors of the solar radiation as well as γ-ray and neutron detectors, which can be successful used also in the outer space have been considered and discussed.

[1] Ya. V. Burak, B. V. Padlyak, V. M. Shevel, Radiat. Eff. Defect Sol. **157**, 1101 (2002).
[2] B. V. Padlyak, V. T. Adamiv, Ya. V. Burak, M. Kolcun, Physica B **412**, 79 (2013).
[3] B. V. Padlyak *et al.*, Opt. Mater. **34**, 2112 (2012).
[4] J. Mikulski *et al.*, Opt. Mater. **38**, 24 (2014).
[5] I. I. Kindrat, B. V. Padlyak, A. Drzewiecki, J. Lumin. **166**, 264 (2015).
[6] I. I. Kindrat, B. V. Padlyak, R. Lisiecki, Opt. Mater. **49**, 241 (2015).
[7] B. V. Padlyak, R. Lisiecki, W. Ryba-Romanowski, Opt. Mater. **54**, 126 (2016).

QUANTUM STEERING GENERATION IN A THREE-QUBIT SYSTEMS

J. K. Kalaga, W. Leoński
Quantum Optics and Engineering Division, Institute of Physics, University of Zielona Góra, Poland

We consider a model of three qubits. In our study we concentrate on a possibility of generation steerable states for different pairs of subsystems. We analyze time evolution of steering parameter based on Cavalcanti inequality [1, 2, 3], and we show that only asymmetric steering can be observed. Next, we discuss mutual time-evolution of the steering parameter and the entanglement measures (such as negativity and concurrence). We show that for steerable two-qubit mixed states the value of negativity is always larger than their concurrence's counterparts.

[1] E. G. Cavalcanti, M. D. Reid, J. Mod. Opt. **54**, 2373 (2007)
[2] E. G. Cavalcanti, P. D. Drummond, H. A. Bachor, M. D. Reid, Opt. Express **17**, 18693 (2009)
[3] E. G. Cavalcanti, Q. Y. He, M. D. Reid, H. M. Wiseman, Phys. Rev. A **84**, 032115 (2011)

MAPPING OF COMPOSITE FERMI-PARTICLES INTO FERMIONS IN MANY-BODY SYSTEMS, AND ITS APPLICATIONS

Yu. A. Mishchenko, A. M. Gavrilik
Bogolyubov Institute for Theoretical Physics of the NAS of Ukraine, Kyiv

The treatment of composite particles as deformed oscillators is of interest in view of its simplicity and encapsulating the internal structure information in the deformation parameters. For two-component composite bosons (like mesons, excitons, Cooper pairs, biphotons, atoms) the operator realization by deformed oscillators was already performed [1] implying a mapping of composite boson states onto deformed oscillator ones. An analogous study of two-component composite fermi-particles, considered herein [4], built of fermion and of either *(i)* usual or *(ii)* deformed boson allows only fermionic operator realization. The latter is explicitly found within the general case of *(i)* and some subcases of *(ii)*. Obtained are the admissible wavefunctions $\Phi^{\mu\nu}_{\alpha}$ involved in the composites' creation $A^{\dagger}_{\alpha} = \sum_{\mu\nu} \Phi^{\mu\nu}_{\alpha} a^{\dagger}_{\mu} b^{\dagger}_{\nu}$ and annihilation A_{α} operators mapping into fermionic ones.

The particular applications of the constructed fermionic realization of two-component fermi-particles to many-body systems are also of interest within the present study. That includes the applicability for different situations (temperatures, interactions and boundness or inter-component entanglement of the composites), calculation of the partition function and other thermodynamic or statistical physics quantities. Among the composite fermi-particles suitable for the above effective description there are three-component complexes like trions or baryons e.g. when two constituents form a bound state modeled by deformed boson. For such modeling of composite constituent boson, the realization from [1]

could be relevant. Besides, the present results may concern cluster nuclei, fermion-boson bound states in correlated fermionic or boson-fermion environments.

Some quantum information aspects are also touched. Similarly to the case of composite bosons [2, 3], the entanglement entropy and purity of the realized composite fermi-particles are found and expressed [4] through the relevant parameters involved in matrices $\Phi_{\alpha}^{\mu\nu}$.

[1] A. M. Gavrilik, I. I. Kachurik, Yu. A. Mishchenko, J. Phys. A: Math. Theor. **44**, 475303 (2011).
[2] A. M. Gavrilik, Yu. A. Mishchenko, arXiv preprint, arXiv:1504.00529 (2015).
[3] A. M. Gavrilik, Yu. A. Mishchenko, Phys. Lett. A, **376**, 1596 (2012)
[4] A. M. Gavrilik, Yu. A. Mishchenko, J. Phys. A: Math. Theor., **46**, 145301 (2013).

SYSTEM OF TWO PARTICLES IN ROTATIONALLY INVARIANT NONCOMMUTATIVE SPACE

Kh. P. Gnatenko
Department for Theoretical Physics,
Ivan Franko National University of Lviv, Ukraine

Noncommutative space has been extensively studied owing to development of String Theory and Quantum Gravity. In the canonical version of three-dimensional noncommutative space coordinates and momenta satisfy the following relations

$$[X_i, X_j] = i\hbar\theta_{ij}, \tag{1}$$

$$[X_i, P_j] = i\hbar\delta_{ij}, \tag{2}$$

$$[P_i, P_j] = 0, \tag{3}$$

here θ_{ij} is a constant antisymmetric matrix, there is the problem of rotational symmetry breaking. In order to preserve the symmetry we propose to generalize the parameter of noncommutativity θ_{ij} to a tensor which is constructed with the help of additional coordinates governed by a rotationally symmetric system. Therefore, the following noncommutative algebra is proposed

$$[X_i, X_j] = i\varepsilon_{ijk}\alpha l_P^2 \tilde{a}_k, \tag{4}$$

$$[X_i, P_j] = i\hbar\delta_{ij}, \tag{5}$$

$$[P_i, P_j] = 0, \tag{6}$$

here α is a dimensionless constant, l_P is the Planck length, $\tilde{a}_i$ are additional coordinates [1]. Algebra (4)-(6) is equivalent to noncommutative algebra (1)-(3) and is rotationally invariant [2].

In general case the coordinates of different particles may satisfy noncommutative algebra with different parameters of noncommutativity. So, there is a problem of describing a composite system in noncommutative space [3]. We study a two-particle system in rotationally invariant noncommutative space (4)-(6). It is shown that the coordinates of the center-of-mass and the coordinates of relative motion satisfy noncommutative algebra with effective tensors of noncommutativity [4].

The hydrogen atom is studied as a two-particle system in rotationally invariant noncommutative space. The corrections to the energy levels of the hydrogen atom are found up to the second order in the parameter of noncommutativity [4].

[1] H. P. Gnatenko, V. M. Tkachuk, Phys. Lett. A **378**, 3509 (2014).
[2] Kh. P. Gnatenko, J. Phys.: Conf. Ser. **670**, 012023 (9 p.) (2016).
[3] Kh. P. Gnatenko, Phys. Lett. A **377**, 3061 (2013).
[4] Kh. P. Gnatenko, V. M. Tkachuk, Ukr. J. Phys. **61**, 432 (2016).

MAGNETICALLY MODIFIED TITANATE NANOTUBES FOR WATER PURIFICATION

M. Marć, M. R. Dudek
Institute of Physics, University of Zielona Góra, Zielona Góra, Poland

Titanate nanotubes are open-ended multi-wall structures with spiral cross section. Preparation procedure is based on the hydrothermal reaction of TiO2 nanopowder and sodium base solution according to Kasuga et al. [1]. The material possesses a broad spectrum of applications which are based on semiconductive and biocompatible properties[2]. Due to high specific surface area it exhibit high adsorption efficiency[3]. In this case magnetic nanoparticles are included onto nanotubes. The potential application in water purification will be discussed.

[1] T. Kasuga, M. Hiramatsu, A. Hoson, T. Sekino, K. Niihara, Langmuir **14**, 3160 (1998).
[2] W. Zhou, H. Liu, R.I. Boughton, G. Du, J. Lin, J. Wang, D. Liu, J. Mater. Chem. **20**, 5993 (2010).
[3] L. Xiong, Y. Yang, J. Mai, W. Sun, C. Zhang, D. Wei, Q. Chen, J. Ni, Chem. Eng. J. **156**, 313 (2010).

VOIDS AS A TEST FOR DARK ENERGY MODELS

B. Novosyadlyj, M. Tsizh
Astronomical observatory, Ivan Franko National University of Lviv, Ukraine

We investigate the possibility to use the observational data on voids as elements of the large-scale structure of the Universe for testing of models of dynamical dark energy. For that we analyse the evolution of cosmological perturbations which leads to the formation of large voids in the distribution of galaxies. We assume that perturbations are spherical and all components of the Universe — radiation, matter and dark energy — are continuous media with ideal fluid energy-momentum tensors, which interact only gravitationally. The equations of evolution of perturbations in the comoving to cosmological background reference frame for every component are obtained from the equations of conservation law and Einstein's ones and are integrated by modified Euler method. The initial conditions are set at the early stage of evolution in radiation-dominated epoch, when the scale of perturbation is much larger than the particle horizon. We investigate the evolution of spatial distributions of density and velocity profiles for every component in the void. We study also the dependences of the void observational parameters on the parameters of dark energy model, in particular, on effective speed of sound and equation of state parameter. We show that ratio of the amplitudes of spatial distributions of peculiar velocity of galaxies and matter density perturbation in voids is sensitive to equation of state parameter of dark energy and weak depends on amplitude and density profile of initial perturbation.

PENNING TRAP WITH AN INCLINED MAGNETIC FIELD

Yu. Yaremko[1], *M. Przybylska*[2], *A. J. Maciejewski*[2]
[1]Institute for Condensed Matter Physics, NAS of Ukraine, Lviv
[2]Institute of Astronomy, University of Zielona, Zielona Góra, Poland

Modified Penning trap with a spatially uniform magnetic field **B** inclined with respect to the axis of rotational symmetry of the electrodes is considered. The inclination angle can be arbitrary. Canonical transformation of phase variables transforming Hamiltonian of considered system into a sum of three uncoupled harmonic oscillators is found. We determine the region of stability in space of two parameters controlling the dynamics: the trapping parameter κ and the squared sine of the inclination angle ϑ_0. If the angle ϑ_0 is smaller than 54 degrees, a charge occupies finite spatial volume within processing chamber. A rigid hierarchy of trapping frequencies is broken if **B** is inclined at the critical angle: the magnetron frequency reaches the corrected cyclotron frequency while the axial frequency exceeds them. Apart from this resonance we reveal the family of resonant curves in the region of stability.

In the relativistic regime the system is not linear. We show that it is not integrable in the Liouville sense. The averaging over the fast variable allows to reduce the system to two degrees of freedom. An analysis of the Poincaré cross-section of the averaged systems shows the regions of effective stability of the trap.

ENTANGLEMENT OF THE SPIN CHAIN WITH LONG-RANGE INTERACTION

A. R. Kuzmak
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

The quantum evolution of a spin-1/2 chain with the long-range Ising interaction is considered. The maximally entanglement 'Schrödinger cat' state can be achieved during such an evolution [1, 2]. Using the expression for the geometric measure of entanglement from [3] we investigate the entanglement of one spin with the remaining spins of chain. Also the entanglement of the system with the large number of spins is examined.

[1] K. Molmer, A. Sorenses, Phys. Rev. Lett. **82**, 1835 (1999).
[2] D. Leibfried *et al.*, Nature **438**, 639 (2005).
[3] A. M. Frydryszak, V. M. Tkachuk, arXiv:quant-ph/1211.6472v2 (2015).

NONLINEAR WAVE EQUATION FOR SHALLOW WATER PROBLEM WITH NONFLAT BOTTOM OF THE CONTAINER

A. Karczewska[1], *P. Rozmej*[2], *Ł. Rutkowski*[2]
[1]Faculty of Mathematics, Computer Science and Econometrics, Zielona Góra, Poland
[2]Faculty of Physics and Astronomy, University of Zielona, Zielona Góra, Poland

One of the methods of derivating wave equations for shallow water problem is expanding velocity potential into a series with respect to small parameters α, β and δ and solving *Euler equations* renormalized to nondimensional variables with boundary conditions in the first and higher degree of approximations for those parameters. For nonrotational flow of incompressible fluid we have

$$\beta\phi_{xx} + \phi_{yy} = 0, \quad \text{dla} \quad 0 < y < 1 + \alpha\eta(x,t) \tag{7}$$

$$\frac{1}{\beta}\phi_y - (\alpha\eta_x\phi_x + \eta_t) = 0, \quad \text{dla} \quad y = 1 + \alpha\eta(x,t) \tag{8}$$

$$\phi_t + \frac{1}{2}(\alpha\phi_x^2 + \frac{\alpha}{\beta}\phi_y^2) + \eta = 0, \quad \text{dla} \quad y = 1 + \alpha\eta(x,t) \tag{9}$$

$$\phi_y = \beta\delta h_x, \quad \text{dla} \quad y = \delta h(x), \tag{10}$$

with velocity potential in the form

$$\begin{aligned}\phi(x,y,t) &= \phi^{(0)} + y\beta\delta\left(h\phi_x^{(0)}\right)_x - \frac{1}{2}y^2\beta\phi_{2x}^{(0)} - \frac{1}{6}y^3\beta^2\delta\left(h\phi_x^{(0)}\right)_{3x} \\ &+ \frac{1}{24}y^4\beta^2\phi_{4x}^{(0)} + \frac{1}{120}y^5\beta^3\delta\left(h\phi_x^{(0)}\right)_{5x} + \frac{1}{720}y^6\beta^3\phi_{6x}^{(0)},\end{aligned} \tag{11}$$

with $\alpha = \frac{a}{H}$, $\beta = \frac{H^2}{L^2}$ and $\delta = \frac{a_h}{H}$ where a is amplitude of disturbance, a_h amplitude of bottom elevation, H mean depth of the container and L corresponds with wave length.

First degree of aproximation is the *KdV equation* but it is derived with assumption of flat bottom. To be able to examine influence of the bottom's shape on the propagation of the disturbance, one must take higher order terms into the consideration. As far as the KdV equation has analitical solutions

(solitons, multi solitons and periodic cnoidal waves), higher order equatios demands numerical methods to be examined.

While examining equations of second order we've observed effects that where not observed in first order aproximation. Some of them have been seen even in flat bottom cases, but other are strictly related to changes of bottom's shape.

DEPENDENCE OF THE EVOLUTION OF 3-COMPONENT SUPERCLASTER SIZE HALO ON DARK ENERGY PARAMETERS

M. Tsizh, B. Novosyadlyj, Yu. Kulinich
Astronomical observatory, Ivan Franko National University of Lviv, Ukraine

We study the evolution of spherical density and velocity perturbations in 3-component (radiation, matter and quintessential dark energy) Universe on superclaster scales. The matter component in dominant in such forming halo at all stages of evolution, exept the earliest one, when radiation component prevails. All component are described in perfect fluid approach with barotropic equation of state. The equation that govern the evolution are conservation and Einstein's ones. The non-linear terms of second order are kept in them to make them more accurate, they also include terms that take into account Silk damping in radiation component. We solve this system of equations numerically. Solution of the system of partial differential equation by modified Euler method gives the evolution of spatial distribution of perturbation in spherical halo. Initial conditions are set at the early stage of evolution in radiation-dominated epoch, when the scale of perturbation is mush larger than particle horizon. Several initial profiles of distributions of densities are considered.

The results show differences in the behaviour of the perturbation of the systems with different values dark energy parameters (equation of state parameter and effective speed of sound of dark energy).

TWO SIMPLE PENDULA INTERACTING HARMONICALLY

W. Szumiński
Institute of Physics, University of Zielona Góra, Poland

We consider the system of two simple pendula that interact by elastic forces according to the Hoke's law in the presence of gravity field. The analysis of global dynamics by means of Poincare sections shows the large chaotic regions, which suggest that the system is not integrable. We give the analytical proof of this fact by means of the Morales–Ramis theory that connect the integrability with properties of differential Galois group of variational equations obtained by the linearization of the system along a particular non-equilibrium solution.

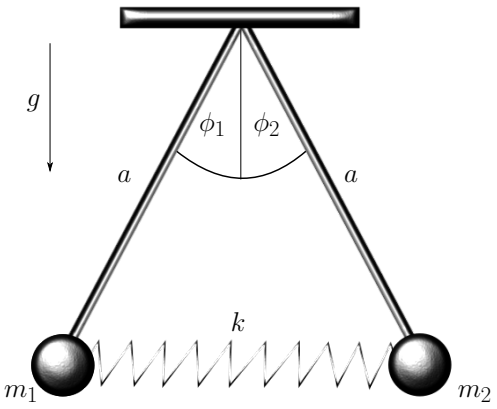


Figure 1: Geometry of the system,

SINGULAR POTENTIALS IN GENERAL CASE OF DEFORMED SPACE WITH MINIMAL LENGTH

M. I. Samar
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

We study deformed Heisenberg algebra leading to minimal length in the general case, when the right-hand side of it is some function of momentum.

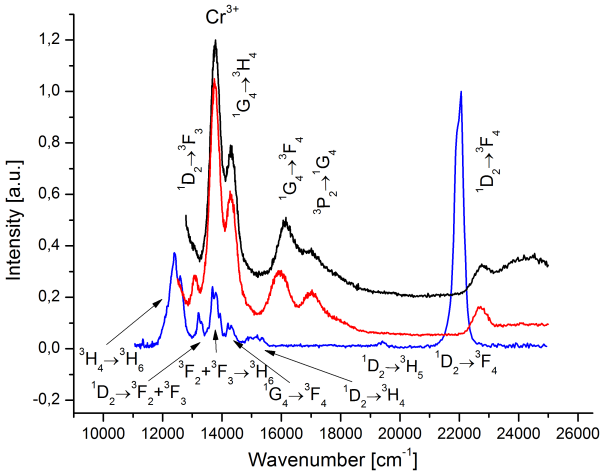
We present a definition of the inverse of position operator which is linear and two-sided [M. I. Samar and V. M. Tkachuk, arXiv:1602.05905]. Our proposal is based on the functional analysis of the position operator. Using this definition 1D Coulomb-like problem is studied and the energy spectrum and the eigenfunctions are found exactly. We analyze the energy spectrum for different partial cases of deformation function and find that the correction caused by the deformation highly depends on the type of the deformation function.

The problem of the attractive inverse square potential in 1D quantum mechanics with a generalized uncertainty relation is studied. Using the momentum representation, we obtain the kernel of the inverse square potential. We solve analytically bound states equation and discuss in detail the bound states spectrum for a specific forms of the generalized uncertainty relation.

$LaAlO_3$: Tm^{3+} SPECTROSCOPIC PROPERTIES DEPENDENCY OF CRYSTAL INTERNAL STRUCTURE

D. Sztolberg[1], *B. Brzostowski*[2], *P. Dereń*[2]
[1]Institute of Low Temperatures and Structure Researches, Wrocław Poland
[2]Institute of Physics, University of Zielona Góra, Zielona Góra, Poland

$LaAlO_3$ powders doped with 0.5 wt%, 3 wt%, 5 wt% and 7 wt% Tm^{3+} ions and $LaAlO_3$ singlecrystal doped with 0.5 wt% was invesitgated. Nanocrystals were prepered with Pechini method and monocrystal was obtained wityh Czochralski method. Powders were syntehsized in two temperatures 800C and 1100C. Absorption, emission and emission decay times were measured in room-temperature (300K) in the IR, visible and near UV range. The synthesis temperature dependency of measured spectroscopic properties and the impact of the internal structure on them are investigated.



[1] Z. Li, S. Zhang, W. E. Lee, J. Europ. Soc. **27**, 3201 (2007).
[2] R. Ianoş, R. Lazău, S. Bocrăncescu, R. Băbuţă, Ceramics Int. **40**, 7561 (2014).
[3] A. Gocalińska, P. J. Dereń, P. Głuchowski, Opt. Material **30**, 680 (2008).
[4] J. Guifang, H. Yihua, Physica B: Condensed Matter **415**, 1 (2013).
[5] F. Benz, A. Gonser, J. Lum. **145**, 855 (2014).

PROBING THE IONIZED INTERSTELLAR MEDIUM USING RADIO PULSAR SIGNALS

W. Lewandowski
Gil Institute of Astronomy, University of Zielona Góra, Poland

When propagating through the interstellar medium the radio signals from pulsars are affected by a few phenomena occurring in the ionized matter. We see the evidence for the interstellar dispersion of radio waves, the scattering as well as the interstellar scintillations. While for the majority of pulsar astronomers these effects are a nuisance that has to be removed from the data to be able to study these objects, they can be also a great source of information about the ionized fraction of the interstellar medium - its physical properties, turbulent nature and distribution within the Milky Way galaxy.

JETS IN THE QUIET SOLAR PHOTOSPHERE

M. Stodilka [1], *A. Sukhorukov* [1,2]
[1]Astronomical observatory, Ivan Franko National University of Lviv, Ukraine
[2]Main Astronomical observatory of NAS of Ukraine, Kyiv

Observations with high spatial resolution (IMaX / SUNRISE and HINODE) showed the presence of jets inside some granules and intergranular layers in a quiet atmosphere of the Sun. But the cause of these phenomena is not found yet. It is unknown whether this is a movement inside of the magnetic loops, or result of magnetic field lines reconnection, or convective collapse. To study such active centers in a quiet atmosphere of the Sun, we used data of 2D observations of the Sun with high spatial resolution (Dutch DOT telescope) in the line FeI λ 5576 A. The formation depths of the line cover photospheric layers of the solar atmosphere. By solving the inverse radiative transfer problem using observational data, we reconstructed the physical conditions (temperature, pressure, density, radial velocity) in the inhomogeneous solar atmosphere; horizontal velocity field was received using the equations of hydrodynamics. In the intergranular areas we revealed small-scale compact formations of increased temperature - one side directed and oposite directed photosphere jets. In the jets the substance goes down or goes up and decreases with height. In the lower photosphere layers the speed of the substance is commensurable with the sound speed. We made an analysis of horizontal velocities near active centers (jets). We also constructed models of observed jets in the quiet photosphere of the Sun.

WETTING IN PRESENCE OF ELECTRIC FIELD: CLASSICAL-DENSITY-FUNCTIONAL-THEORY STUDY FOR MODEL SYSTEM

V. Myhal[1], *O. Derzhko*[2,1,3]
[11]Department for Theoretical Physics, Ivan Franko National University of L'viv, Ukraine
[2]Institute for Condensed Matter Physics, NAS of Ukraine, Lviv
[3]Abdus Salam International Center for Theoretical Physics, Trieste, Italy

We discuss the effect of an external electric field on the wetting of a solid surface by liquid. To this end we use a model of two-level-atom fluid, for which the changes in interatomic interactions due to the presence of the field can be found using a quantum-mechanical perturbation theory. Constructing the grand potential functional, we perform the standard calculations of the contact angle. Switching on of the electric field may increase noticeably the contact angle and yield a wetting-drying transition.

SQUASHED TOPOLOGICAL KALUZA-KLEIN BLACK HOLE SOLUTION

M. M. Stetsko
Department for Theoretical Physics, Ivan Franko National University of L'viv, Ukraine

A new static solution of Einstein-Maxwell equations is considered. Properties of such space-time are investigated. Comparison with some topological solutions is made.

THERMODYNAMIC QUANTITIES OF THE LOW-DENSITY GAS IN THE WEAKLY NONEQUILIBRIUM HEAT-CONDUCTION STEADY STATE IN THE LINEAR TEMPERATURE PROFILE APPROXIMATION

Y. A. Humenyuk
Institute for Condensed Matter Physics, NAS of Ukraine, Lviv

Heat-conduction properties of macroscopic systems in the weakly nonequilibrium states are well described by the linear Fourier law, which originates from experimental facts and phenomenological considerations. It is also applicable to the stationary cases when relaxation processes are absent.

As to thermodynamic features of the system in the heat-conduction steady state, e.g., pressure, internal energy, and entropy, the problem of determination of these quantities stays at the outset of solution. [1–5]. In most cases, arguments borrowed from kinetic theory or nonequilibrium statistical mechanics are used. In recent works [6–8] the problem of the nonequilibrium pressure in heat-conducting stationary liquids is treated by the mode-coupling theory. Such approaches do not correspond to general methodological grounds of thermodynamics which is phenomenological by its nature. Satisfactory constructions are lacking even for simple models like a low-density gas. Consequently, it is desirable to have an example of a simple phenomenological description of the thermodynamic quantities, which would be a reasonable approximation and give proper thermodynamic estimates.

To this end, we use as before [9] the continuous media model and accept for the weakly nonequilibrium case the linear form of the temperature profile. Such a "minimal" approximation, though neglects the higher gradients, but remains in accordance with the Fourier law. It provides simple analytical estimates for the thermodynamic quantities of the low-density gas. The baric and caloric equations of the heat-conduction steady state as well as expressions for isothermal compressibilities and heat capacities are derived. The entropy obtained is shown to satisfy the second law of thermodynamics. The approximation of the displaced linear profile is proposed which is aimed to take into account nonlinearity of the space dependence of the temperature.

These results can be a test instrument for more realistic and detailed approaches and approximations.

[1] R. E. Nettleton, J. Chem. Phys. **93**, 8247 (1990).
[2] D. Jou, J. Casas-Vázquez, G. Lebon, Extended irreversible thermodynamics, Springer, Berlin, 2001.
[3] H.-D. Kim, H. Hayakawa, J. Phys. Soc. Jpn. **72**, 1904 (2003); **72**, 2473 (2003).
[4] S.-i. Sasa, H. Tasaki, J. Stat. Phys. **125**, 126 (2006).
[5] G. P. Morriss, D. P. Truant, Phys. Rev. E **87**, 062144 (2013).
[6] T. R. Kirkpatrick, J. M. Ortiz de Zárate, J. V. Sengers, Phys. Rev. Lett. **110**, 235902 (2013).
[7] T. R. Kirkpatrick, J. M. Ortiz de Zárate, J. V. Sengers, Phys. Rev. E **89**, 022145 (2014).
[8] T. R. Kirkpatrick, J. M. Ortiz de Zárate, J. V. Sengers, Phys. Rev. E **93**, 012148 (2016).
[9] Y. A. Humenyuk, Ukr. J. Phys. **61**, 400 (2016).

FIELD EQUATIONS IN SPACE WITH SPIN NONCOMMUTATIVITY OF COORDINATES

V. M. Vasyuta
Department for Theoretical Physics, Ivan Franko National University of L'viv, Ukraine

Noncommutative coordinates appear in string theory [1] and quantum gravity [2]. The most popular canonical noncommutativity $[X^\mu, X^\nu] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is a constant antisymmetric matrix, has some problems. The biggest one is a violation of the rotational invariance (or more widely Lorentz-invariance). One of the possible ways of building a rotational invariant noncommutativity is to use some functions of spin operators instead of constants in the right-hand side of coordinate commutators.

We propose a relativistic Lorentz-invariant noncommutativity by shifting position operators by the Dirac gamma matrices $X^\mu = x^\mu + i\theta\gamma^\mu$ [3]. The corresponding coordinate commutator reads $[X^\mu, X^\nu] = 2i\theta^2\sigma^{\mu\nu}$ and recalls the canonical noncommutativity with replacing $\theta^{\mu\nu} \to 2\theta^2\sigma^{\mu\nu}$. Using the Weyl ordering of noncommutative position operators, we build an analogue of the Moyal star product for the proposed noncommutative algebra.

The Lagrange function of the electromagnetic field in the space with spin noncommutativity is found. In such a space, electromagnetic field becomes non-Abelian. A gauge transformation law for this field is also obtained.

Exact equations of noncommutative electromagnetic field are derived from the least action principle. These equations are nonlinear and contain derivatives of all orders. Nevertheless, the structure of these equations allows quite simple perturbative consideration of them. Moreover, we find an exact solution of electromagnetic wave propagating in a constant electric and magnetic fields.

[1] N. Seiberg, E. Witten, JHEP **09**, 032 (1999).

[2] S. Doplicher, K. Fredenhagen, J. E. Roberts, Phys. Lett. B **331**, 39 (1994).

[3] V. M. Vasyuta, V. M. Tkachuk, arXiv:1604.07448 [hep-th].

ON STOCHASTIC SECOND ORDER KORTEWEG – DE VRIES TYPE EQUATIONS

M. Szczeciński

Faculty of Mathematics, Computer Science and Econometry,
and Faculty of Physics and Astronomy, University of Zielona Góra, Poland

Nonlinear wave equations with soliton solutions play an important role in contemporary physics and mathematics. The simplest equation of that type is famous Korteweg-de Vries (KdV) equation. The KdV equation was originally derived for shallow water problem, that is, for gravitational waves on the surface of a shallow water. The set of Euler equations for irrotational motion of the fluid with proper boundary conditions is difficult to solve. However, in a limit of long small waves and shallow water it is possible to simplify Euler equations and obtain approximate nonlinear wave equations. The equation obtained in the first order approximation was obtained by Korteweg and de Vries in 1985 [1] and became a prototype of nonlinear wave equations. Recently more advanced second order approximation named *extended KdV* [2] or KdV2 [3, 4] attracted growing interest. In [3] the authorsextended the derivation of the KdV2 equation for the case when the bottom of the fluid is not flat. In [4] we demonstrated in numerical simulations for stochastic KdV2 equations that both soliton and cnoidal solutions are very robust with respect to stochastic forces.

In the presentation, which is based on the paper [5], we give sufficient conditions for the existence and uniqueness of mild solution of a stochastic KdV2-type equation. The proof uses approach and estimates from papers [6, 7, 8]. Moreover, we supply conditions for exponential stability and stability in probability of the mild solution mentioned above. For obtaining stability results we used approach developed in [9].

[1] D. J. Korteweg, H. de Vries, Phil. Mag. **39**, 422 (1895).

[2] T. R. Marchant, N. F. Smyth, J. Fluid Mech. **221**, 263 (1990).

[3] A. Karczewska, P. Rozmej, E. Infeld, Phys. Rev. E **90**, 012907 (2014).

[4] A. Karczewska, M. Szczeciński, P. Rozmej, B. Boguniewicz, Comput. Meth. Phys. Tech. (2016), in print.

[5] A. Karczewska, M. Szczeciński, (2016), submitted.

[6] A. de Bouard, A. Debussche, J. Funct. Anal. **154**, 215 (1998).

[7] C.E. Kenig, G. Ponce, L. Vega, J. Amer. Math. Soc. **4**, 323 (1991).

[8] F. Linares, G. Ponce, *Introduction to Nonlinear Dispersive Equations* (Universitext, Springer, 2009).

[9] K. Liu, *Stability of infinite dimensional stochastic differential equations with applications* (Chapman & Hall, 2006).

B-MODE POLARIZATION FROM THE PRIMORDIAL GRAVITATIONAL WAVES: THEORY AND OBSERVATIONS

O. Sergijenko, B. Novosyadlyj
Astronomical observatory, Ivan Franko National University of Lviv, Ukraine

We review the theory of generation of the primordial tensor perturbations (gravitational waves) during cosmological inflation (considering various scenarios) and their imprint on the B-mode polarization of CMB. We also review the corresponding observational projects and their results.

We present the observational constraints on contribution from the tensor mode of perturbations in the models with dynamical dark energy which is assumed to be a classical scalar field with the barotropic equation of state. The used datasets include Planck data on CMB anisotropy and lensing, BICEP2/Keck Array data on B-mode polarization, BAO from SDSS and 6dFGS, power spectrum of galaxies from WiggleZ, weak lensing from CFHTLenS and SN Ia data from the JLA compilation.